An Activity-Based Approach to
Technology Integration in the Mathematics Classroom
January 19, 2009

Neal Grandgenett, Ph.D. (Corresponding Author)
College of Education
University of Nebraska at Omaha
Omaha, Nebraska 68182
ngrandgenett@unomaha.edu
O - 402.554.2690
F - 402.554.3744

Judith Harris, Ph.D.
School of Education
College of William and Mary
P.O. Box 8795
Williamsburg, Virginia 23187
judi.harris@wm.edu
O - 757.221.2334
F - 757.221.2988

Mark Hofer, Ph.D.
School of Education
College of William and Mary
P.O. Box 8795
Williamsburg, Virginia 23187
mark.hofer@wm.edu
O - 757.221.1713
F - 757.221.2988

A manuscript submitted to the Journal of Mathematics Education Leadership
(National Council of Supervisors of Mathematics)

An Activity-Based Approach to Technology Integration in the Mathematics Classroom

> Tim was so learned, that he could name a horse in nine languages.
> So ignorant, that he bought a cow to ride on.
>
> Benjamin Franklin, 1914, p.54

Integrating Technology into Mathematics Instruction

Becoming a mathematics teacher today can be a challenging endeavor, requiring teachers to learn difficult content and specialized pedagogies, as well as new technological tools and techniques. Pre-service teachers at the secondary level are faced with programs of study that often begin with Calculus, and include abstract topics such as non-Euclidean geometry, discrete mathematics, and modern algebra (NCTM, 2000; U.S. Dept. of Education, 2005). With increased federal mandates in mathematics, today's elementary teachers are also faced with significant mathematics content, involving topics such as number sense, geometry, and probability (NCTM, 2006). At the same time, teachers of mathematics at all levels are encouraged to consider relatively sophisticated strategies for instruction, such as problem-based learning, constructivism, and scaffolding (NCTM, 1999; Fuson, Kalchman, & Bransford, 2005). Technologies such as graphing calculators, symbolic processing programs, mathematical simulations and cross-discipline instructional tools such as robotics kits are becoming ever more available to teachers to aid instruction (Heid, 2005). Such a broad variety of instructional responsibilities and opportunities makes mathematics educators worry about furthering the "mile wide and inch deep" instructional challenges that have plagued mathematics instruction in the past (Kaser, Bourexis, Loucks-Horsley, & Raizen, 1999; Adams, Brower, Hill & Marshall, 2000; NCTM 2005). As Mr. Franklin admonished, how do we help teachers not to become so learned in mathematical concepts, yet so ignorant in modern mathematics instruction, that they essentially "buy a cow to ride on" and thus poorly combine content, pedagogy, and technology to form classroom activities that are instructionally unsound?

One answer may lie in the ways that we consider how content, pedagogy and technology might combine to plan effective instruction in today's quickly evolving mathematics classroom. When recommending ways to teach mathematics, it is important to realize that the discipline is changing rapidly as mathematics educational technologies evolve. This, in turn, is further challenging mathematics teachers to keep up with new content and instructional strategies (Heid, 2005; Peterson, 1988; Sinclair & Crespo, 2006). The concept of fractals is a good example. Computational technologies have helped mathematicians to represent and apply fractional dimensions, investigating and modeling real-life phenomena such as lightening strikes, plant growth, cloud formation, coastline erosion, and blood circulation. Yet the integration of fractals concepts into textbooks and coursework is comparatively new to mathematics teachers, and can be difficult to understand or explain without the use of technology (Falconer, 2003). Technology use is similarly changing the mathematics of statistics, graphing, geometry, matrices, and probability, to name just a few. Mathematics teachers are encouraged to use a wide range of educational technologies, such as graphing calculators, symbolic processing programs, spreadsheets and simulations, to help their students to learn about such technology-related topics (Heid, 2005; Rosen, 1999). Given the proliferation of new mathematics content, new instructional strategies, and new mathematics-based educational technologies, how can mathematics teachers make optimal choices to support students' learning?

One new framework, called technological pedagogical content knowledge or TPACK (Koehler, & Mishra, 2008) describes the interconnected and interdependent content, pedagogy, and technology knowledge that teachers must have to make and enact such choices. TPACK is built upon Shulman's (1986, 1987) notions of pedagogical content knowledge: the knowledge necessary to teach particular curriculum content, which incorporates both disciplinary and general pedagogical knowledge TPACK designates knowledge of educational technologies – especially how to use these rapidly proliferating tools and resources instructionally, and in varying educational contexts – as teachers' knowledge that requires deliberate examination and development. Mathematics teachers who have well-developed TPACK may demonstrate this knowledge through using an imaginative mix of mathematical content, pedagogies, and technologies within their lessons (Grandgenett, 2008). In some ways, mathematics teachers have a bit of a head start on TPACK development, since the profession's integration of instructional technology to date, when compared with other disciplines, has been relatively strong. For example, the use of graphing calculators in high school mathematics classes has been suggested by several authors as one of the best examples of the successful integration of technologies into teaching and learning (Fuson et. al, 2005; Kaser et. al, 1999; Reece, Dick, Dildine, Smith, Storaasli, Travers, Wotal, & Zygas, 2005). Technology-based applications like *Geometer's Sketchpad* and *Excel*, and Web-based resources like the *National Library of Virtual Manipulatives* are relatively common and well embraced in today's mathematics classroom (Heid, 2005).

However, current integrations of digital technologies, such as graphing calculators or *Excel*, only scratch the surface of the educational opportunities that these tools and resources make possible in mathematics instruction. Innovative software programs such as *Inspire Data* or the newly enhanced *Mathematica*, or new technologies such as robotics and global positioning systems (GPS) are providing exciting opportunities for the learning of mathematics, as well as creating challenges to both educators and teacher preparation programs. When considering the possibilities for effective technology use in the mathematics classroom, there appear to endless possibilities. Yet, if one considers the learning activities that a mathematics teacher might *typically* plan,, and the technologies that are generally seen to be useful within that context, a more limited list of possibilities would probably be generated. We suggest that providing teachers with a more comprehensive list of possible mathematics learning activities, along with the specific educational technologies that can best support each, will help both teachers and teacher educators to better integrate educational technologies into mathematics instruction. We believe that regular use of such a taxonomy could contribute significantly to both teachers' TPACK development, and their professional development overall..

We have attempted to generate and categorize this taxonomy of mathematics learning activities, as well as to identify some useful technologies that can support each activity. The taxonomy was generated through collaborative discussion, after a careful review of the technology-based activities represented in the last five years of articles published in the three practitioner journals published by the National Council of Teachers of Mathematics: *The Mathematics Teacher*, *Mathematics Teaching in the Middle School*, and *Teaching Children Mathematics*. In all, more than 180 journal issues were examined. A teacher planning a lesson for a particular mathematical topic or concept can review this taxonomy, selecting several learning activities to combine to structure a lesson, unit, or project plan. We have identified 31 distinct mathematics learning activity types, and acknowledge that some may need to be added, removed, edited, or combined. Considering the rapid evolutions of mathematics, mathematics education, and educational technologies, the list will no doubt also grow and evolve along with advancements in the discipline. Our taxonomy is presented as a beginning point for others to consider and to refine. (We have set up a wiki at http://activitytypes.wmwikis.net/ to facilitate this process.).

As mentioned above, we hope that by creating and sharing this taxonomy of mathematics learning activity types, we can assist the development of “TPACK in action” for mathematics teachers. Even at the most pragmatic level, the taxonomy should be useful to teachers as they select instructional strategies and tools to help students to meet particular curriculum content standards. The individual activity types represent possibilities for teacher instruction, but are conceptualized in terms of students’ actions, focusing upon what a teacher might help students to *do* during mathematics instruction. For example, a teacher brainstorming ways to teach the concept of algebraic slope might consider the use of a strategy and a related technology that they might not have initially contemplated, such as “interpreting a phenomenon mathematically” by having students drive an electronic car up different sloped ramps, and then using an interactive graphing program to represent the changing equation of the ramp slope.

Mathematics Learning Activity Types

As mentioned, the purpose of presenting a learning activity types taxonomy for mathematics is to introduce the full range of possible student learning activities for teachers to consider when building lessons that effectively integrate technology, pedagogy, and content. In doing so, we attempt to scaffold teachers’ thinking about how to best structure learning activities, best support those activities with educational technologies, and to spark creativity during instructional planning.

Essentially, these mathematics activity types are designed to be *catalysts* to thoughtful and creative instruction by teachers. We have conceptualized seven genres of activity types for mathematics that are derived from the NCTM's process standards. To encourage active engagement by all students, these activity types are expressed using active words to represent the pursuit of a dynamic and student-centered learning environment: Consider, Practice, Interpret, Produce, Apply, Evaluate, and Create. Most of these words are drawn directly from the NCTM standards. Each of the seven genres is presented in a separate table below that names the activity types included in that genre, defines them briefly, and then provides some example technologies that could support students' learning in each activity well.

The "Consider" Activity Types

When learning mathematics, students are often asked to consider new concepts or information. This request is a familiar one to both students and teachers. Yet, although such learning activities can be very important contributors to student understanding, the "Consider" activity types also often produce some of the lowest levels of student engagement, and are manifested typically using a comparatively direct presentation of foundational knowledge.

Table 1: The "Consider" Activity Types

Activity Type	**Brief Description**	**Example Technologies**
Attend to a Demonstration	The student gains information from a teacher or student presentation, videoclip, animation, interactive whiteboard or other display media.	Powerpoint, YouTube, document camera, interactive whiteboard, videoconferencing, or other display media
Read Text	The student extracts information from textbooks, or other written materials, in either print or digital form.	Electronic textbooks, websites (i.e. the Math Forum), informational .pdfs
Discuss	The student discusses a concept or process with a teacher, other students, or an external expert.	Ask-an-expert sites (e.g., Ask Dr. Math), online discussion groups, videoconferencing
Recognize a Pattern	The student examines a pattern presented and attempts to more fully understand the pattern.	Graphing calculators, virtual manipulative sites (e.g., the National Library of Virtual Manipulatives), spreadsheets
Investigate a Concept	The student explores or investigates a concept (such as fractals), perhaps by use of the Internet or other research-related resources.	Web searching, informational databases (Wikipedia), virtual worlds (Second Life), simulations
Understand or Define a Problem	The student strives to understand the context of a stated problem or to define the mathematical characteristics of a problem.	Web searching, concept mapping software, ill-structured problem media (i.e. Jasper Woodbury)

The "Practice" Activity Types

In the learning of mathematics, it is often important for students to be able to practice computational techniques or other algorithm-based strategies, so that these skills can be automated for later and higher-level mathematical application. Some educational technologies can be used to assist these processes. The table below offers both the range of practice-based learning activities and examples of how technology can assist in their implementation.

Table 2: The "Practice" Activity Types

Activity Type	**Brief Description**	**Example Technologies**

Do Computation	The student undertakes computation-based strategies using numeric or symbolic processing.	Scientific calculators, graphing calculators, spreadsheets, Mathematica
Drill and Practice	The student rehearses a mathematical strategy or technique, and perhaps uses computer-aided repetition and feedback in the practice process.	Mathblaster drill and practice software, online textbook supplements, online homework help websites (WebMath).
Solve a Puzzle	The student carries out a mathematical strategy or technique within the context of solving an engaging puzzle, which may be facilitated or posed by the technology.	Virtual manipulatives, Web-based puzzles (magic squares), brainteaser Web sites (CoolMath)

The "Interpret" Activity Types

In the discipline of mathematics, concepts and relationships can be quite abstract, representing, at times, a bit of a mystery to students. They often need to spend time deducing and explaining these relationships to internalize them. Educational technologies can be used to help students investigate concepts and relationships more actively, and to assist with interpretion of what they observe. This table displays activity types that can support such interpretive processes, and provides examples of available technologies that can be used to support their formation.

Table 3: The "Interpret" Activity Types

Activity Type	Brief Description	Example Technologies
Pose a Conjecture	The student poses a conjecture, perhaps using dynamic software to display relationships.	Dynamic geometry software (Geometer’s Sketchpad), widgets (Explore Learning), e-mail
Develop an Argument	The student develops a mathematical argument related to why they think that something is true. Technology may help to form and to display that argument (e.g. a proof).	Concept mapping software (Inspiration), presentation software, blogs, specialized word processing software (Theorist), e-mail
Categorize	The student attempts to examine a concept or relationship in order to categorize it into a set of known categories.	Database software (Microsoft Access), online databases, concept mapping software, drawing software
Interpret a Representation	The student explains the relationships apparent from a mathematical representation (table, formula, chart, diagram, graph, picture, model, animation, etc.).	Data visualization software (Inspire Data), 2D and 3D animations, video (iMovie), Global Positioning Devices (GPS), engineering visualization software (MathCad)
Estimate	The student attempts to approximate some mathematical value, by further examining relationships using supportive technologies.	Scientific calculator, graphing calculator, spreadsheets, student response systems (Clickers)
Interpret a Phenomenon Mathematically	Assisted by technology as needed, the student examines a mathematics related phenomenon (such as velocity, acceleration, the Golden Ratio, gravity, etc.).	Digital cameras, video, computer-aided laboratory equipment, interactive graphing program, specialized word processing, robotics, electronics kits

The "Produce" Activity Types

When students are actively engaged in the study of mathematics, they can become motivated producers of mathematical works, rather than just passive consumers of prepared materials. Educational technologies can serve as excellent “partners” in this production process, aiding in the refinement and formalization of student products, as well as helping the student to share the fruits of their mathematical labors. The activity types listed below suggest technology-assisted efforts in which this occurs.

Table 4: The "Produce" Activity Types

Activity Type	**Brief Description**	**Example Technologies**
Do a Demonstration	The student demonstrates a topic or concept to show their understanding of a mathematical idea or process. Technology may assist in the development or presentation of the product.	Interactive whiteboard, video (YouTube), document camera, presentation software, podcasts
Generate Text	The student produces a report, annotation, explanation, journal entry or document, to illustrate their understanding.	Specialized word processing (Math Type), collaborative documents (Google docs), blogs, online discussion groups
Describe an Object or Concept Mathematically	Technology may assist in the description or documentation process, as the student produces a mathematical explanation of an object or concept.	Engineering visualization software, concept mapping software, specialized word processing, Mathematica
Produce a Representation	The student develops a mathematical representation (table, formula, chart, diagram, graph, picture, model, animation, etc.) using technology for production assistance, if necessary.	Spreadsheet, virtual manipulatives (digital geoboard), spreadsheets, Inspire Data, concept mapping software, graphing calculator
Develop a Problem	The student poses a mathematical problem that is illustrative of some mathematical concept, relationship, or investigative question.	Word processing, online discussion groups, Wikipedia, Web searching, e-mail

The "Apply" Activity Types

The utility of mathematics in the physical world can be found in its authentic applications. Educational technologies can be used to help students to apply mathematics in the world, and to link mathematical concepts to real-world phenomena. The technologies essentially become students’ assistants in their mathematical work, helping them to link mathematical concepts to the realities in which they live.

Table 5: The "Apply" Activity Types

Activity Type	**Brief Description**	**Example Technologies**
Choose a Strategy	The student reviews or selects a mathematics related strategy for a particular context or application.	Online help sites (WebMath, Math Forum), Inspire Data, dynamic geometry/algebra software (Geometry Expressions), Mathematica, MathCAD
Take a Test	The student applies their mathematical knowledge within the context of a testing environment, such as with computer-assisted testing software.	Test-taking software, Blackboard, survey software, student response systems

Apply a Representation	The student applies a mathematical representation to a real life situation (table, formula, chart, diagram, graph, picture, model, animation, etc.).	Spreadsheet, robotics, graphing calculator, computer-aided laboratories, virtual manipulatives (algebra tiles)

The "Evaluate" Activity Types

When students evaluate the mathematical work of others, or self-evaluate their own, they utilize and develop a comparatively sophisticated understanding of mathematical concepts and processes. Educational technologies can become valuable allies in this effort, assisting students in the evaluation process by helping them to undertake concept comparisons, test solutions or conjectures, and/or integrate feedback from other individuals into revisions of their own work. The following table lists some of these evaluation-related mathematics learning activities.

Table 6: The "Evaluate" Activity Types

Activity Type	**Brief Description**	**Example Technologies**
Compare and Contrast	The student compares and contrasts different mathematical strategies or concepts, to see which is more appropriate for a particular situation.	Inspiration, Web searches, Mathematica, MathCad
Test a Solution	The student systematically tests a solution, and examines whether it makes sense based upon systematic feedback, which might be assisted by technology.	Scientific calculator, graphing calculator, spreadsheet, Mathematica, Geometry Expressions
Test a Conjecture	The student poses a specific conjecture and then examines the feedback of any interactive results to potentially refine the conjecture.	Geometer Sketchpad, statistical packages (e.g/, SPSS, Fathom), online calculators, robotics
Evaluate Mathematical Work	The student evaluates a body of mathematical work, through the use of peer or technology-aided feedback.	Online discussion groups, blogs, Mathematica, MathCad, Inspire Data

The "Create" Activity Types

When students are involved in some of the highest levels of mathematics learning, they are often engaged in very creative and imaginative thinking processes. Albert Einstein once implied that imagination was more important than knowledge when it came to mathematics (Priwer & Phillips, 2003). It is said that this idea was consistent with his strong belief that mathematics is a very inventive, inspired, and imaginative endeavor. Educational technologies can be used to help students to be creative in their mathematical work, and to assist other students' mathematics learning. The activity types below represent these creative elements and processes in students' mathematical learning and interaction.

Table 7: The "Create" Activity Types

Activity Type	**Brief Description**	**Example Technologies**

Teach a Lesson	The student develops and delivers a lesson on a particular mathematics concept, strategy, or problem.	Presentation software, interactive video, video, podcasts
Create a Plan	The student develops a systematic plan to address some mathematical problem or task.	Concept mapping software, collaborative writing software, MathCad, Mathematica
Create a Product	The student imaginatively engages in the development of a student project, invention, or artifact, such as a new fractal, tessellation, or other creative product.	Word processor, animation tools, MathCad, Mathematica, Geometer Sketchpad
Create a Process	The student creates a mathematical process that others might use, test or replicate, essentially engaging in mathematical creativity.	Computer programming, robotics, Mathematica, MathCad, Inspire Data, iMovie

Combinations of Activity Types

A creative lesson plan by a teacher usually combines two or more activity types into a varied and engaging learning experience. In fact, when learning activities are combined and integrated, they may better resemble the complexity of real-life applications of mathematics and help the problem solver to practice solving problems that are more realistic than the relatively artificial problems often found in textbooks (Checkly, 2006; Fuson et. al, 2005). Combining activity types may also provide opportunities for students to develop more divergent ways of thinking, such as in mathematical modeling work, which uses mathematical language, such as equations, to describe systems (Aris, 1994; Gershenfeld, 1998). Here are a few examples of mathematics learning plans, including a simple combination and two more complex combinations of mathematics activity types.

Example 1: Recognizing and Researching the Fibonacci Series.

A common mathematics topic for teachers to assign for student research in the middle school mathematics classroom is the remarkable Fibonacci Series. This series, in which each term is created by summing the two terms that appear before it (i.e. 1, 2, 3, 5, 8, 13, 21, 34, etc.) is found very commonly in nature, in such items as the spiraled skin of pineapples, the stems of conifer trees, the curved edges in sea shells, and even the family trees of honeybees (Cook, 1979). A simple combination of activity types that might be used by teachers for building student understanding of the Fibonacci Series, is to first ask students to "recognize [the] pattern" (from the “Consider” activity types) by displaying it on a chalkboard or spreadsheet, to see if students can add terms to the sequence correctly. Then the students might "investigate [the] concept" (also from the “Consider” activity types) by doing a Web search on the Fibonacci series to discover where it might be represented in the physical world. Students are often amazed at the many diverse examples of this series that can be found in the natural world.

Example 2: Defining, Representing, and Solving a Paper Folding Problem

An interesting problem that is sometimes posed to elementary students who are studying exponential numbers, is how thick a piece of newspaper might be if it is folded in half, a total of 10 times. To help students to define the problem, they are often given a single large piece of newsprint to attempt the folding. However, the thickness of the folded paper soon creates an impossible situation, and students find that they need to move to computational strategies to solve the problem. They are essentially faced with the rapidly expanding exponential sequence of 1, 2, 4, 8, 16, 32, etc., that doesn't allow them to fold the paper more than 6 or 7 times. At this point, a teacher might encourage students to use a spreadsheet to mathematically "represent" (from the “Produce” activity types) the problem in columns on the spreadsheet, where each fold numerically describes measurements of the growing thickness. Students might then be encouraged to "test a conjecture" (from the “Evaluate” activity types) or solution (also from the “Evaluate” activity types) to this problem, which relates to powers of 2, and eventually determine that the solution is most likely 2 to the 10th power, or 1024 units of thickness. If the teacher realizes that this same problem has been showcased on the television program "MythBusters," s/he could end the lesson by asking students to "attend to" (from the “Consider” activity types) the Mythbusters clip. In this video, the show’s hosts jokingly use a sheet of paper the size of a football field, and modern construction equipment, to see if the size of the paper makes any difference in how many folds they are able to make while trying to solve the problem. (They eventually get to 12.)

Example 3: Interpreting, Producing, and Testing a Garbage Pickup Model

In high school discrete mathematics, a common combination of activities is focused upon modeling how a garbage truck might efficiently move through a system of streets to pick up the garbage each week. A teacher might encourage students to "interpret a representation" (from the “Interpret” activity types) by examining maps of local streets, or perhaps viewing a satellite image of their area using Google Maps. The students could then be asked to “understand or define a problem” and decide upon the parameters for efficient garbage pickup. Typically, students describe various considerations, such as the need to conserve gas by not retracing a route once the truck has already traveled a street. The students are then encouraged to "produce a representation" (from the “Produce” activity types) of the streets as a "network" of line segments for the streets and nodes for the street intersections. They are then asked to "create a plan" (from the “Create” activity types) for an efficient garbage pickup route using this mathematical representation of their neighborhood. Often, students prefer to use some sort of computer-assisted drawing capabilities, such as the drawing utilities in Microsoft Word, or the more sophisticated *MathCad*, to depict the system of nodes and connecting line segments, and to formalize their planning. Soon students also realize that "odd or even" nodes (which are named according to the number of line segments at a street intersection), is also a consideration for planning the most efficient route.

Finally, students might be asked to "compare and contrast" their routes (from the “Evaluate” activity types) by creating some sort of numerical index for their route (perhaps with a spreadsheet chart) that might compute the number of miles traveled and/or the amount of gasoline used. In doing so, they are encouraged to “evaluate [their own and others’] mathematical work” (from the “Evaluate” activity types) to create maximally efficient routes. This particular mathematical challenge illustrates a realistic use of mathematical modeling, while also exemplifying a combination of mathematical learning activity types that encourages flexibility, creativity, and pedagogically appropriate technology use.

Final Thoughts

As in the mathematical modeling process, the discipline of mathematics itself is a very creative, exciting and dynamic endeavor. As defined by Steen, mathematics "involves observing, representing and investigating patterns and relationships in social and physical phenomena and between mathematical objects themselves" (1998, page 16). The mathematics learning activity types presented in this article can hopefully help teachers to better engage and motivate students in their classrooms, perhaps involving these students more fully in the creativity of real-life mathematics, as well as helping them to appreciate the great utility of mathematics in understanding our natural world. Most importantly, if used effectively, the learning activity types might help a teacher to make mathematics instruction more meaningful and relevant, by encouraging students to see authentic applications and growing potential for this important discipline, which is becoming increasingly linked to technology.

If we are to help teachers to develop their TPACK, so that they can combine technology, pedagogy, and content successfully in their classrooms, we will no doubt need to do so via clear, instructionally sound strategies and examples. . When we have separated content, pedagogy, and technology in our pre-service and in-service programs in the past, we have risked teachers' very superficial understanding of the instructional power of their successful combination, with less-than-optimal instructional results. We need instead to carefully and consciously scaffold the development of teachers' TPACK, so that they can make thoughtful and maximally effective instructional choices that combine technology, pedagogy, and content into lessons that are characterized by an engaging and authentic blend of both the theoretical and application perspectives of mathematics.

Such integration can be done, and done well, if we give teachers the support and encouragement they need to be creative designers of classroom instruction. Ben Franklin would no doubt be in agreement with this approach, since he had an uncommon passion for applying knowledge to the world in which he lived, advocating for an educational system that was both philosophically and practically sound. The taxonomy of mathematics learning activity types shared here is a vehicle that can be used to support teachers who are trying to make mathematics more comprehensible and practical for their students, while also building interest and motivation through diverse, engaging and technology-rich learning activities. If we are successful in such efforts -- in the words of Mr. Franklin -- our students will be less likely to "buy a cow to ride on" and instead be able to apply mathematics effectively to real-world situations,, while continuing to grow in their understanding and appreciation of mathematics and its many uses..

References

Adams, K., Brower, S., Hill, D., & Marshall, I. (2000). *The components of effective mathematics and science in middle school: Standards, teaching practices, and* *professional development*. San Marcos, TX: Texas State University (ERIC Document Service No. ED 449032)

Aris, R. (1994). Mathematical Modelling Techniques, New York: Dover. ISBN 0-486-68131-9.

Checkley, K. (2006). "Radical" math becomes the standard: Emphasis on algebraic thinking, problem solving, communication. *Education Update*, newsletter within the Association for Supervision and Curriculum Development, 48 (4), 1-2, 8.

Cook. T.A. (1979). The curves of life. New York: Dover. ISBN 0 486 23701 X.

Falconer, K. (2003). *Fractal geometry: Mathematical foundations and applications.* Chichester West Sussex, England: John Wiley and Sons.

Franklin, B. (1914). *Poor Richard's Almanack.* Waterloo, Iowa: The U.S.C. Publishing Company.

Fuson, K.C., Kalchman, M., Bransford, J.D. (2005). Mathematical understanding: An Introduction. *How students learn: History, mathematics, and science in the classroom*. Committee on How People Learn, A targeted report for teachers from the National Research Council, Donovan, M.S. and Bransford, J.D. (Eds). Washington, DC: The National Academies Press.

Gershenfeld, N. (1998). The Nature of Mathematical Modeling, Cambridge University Press, (1998) ISBN 0-521-57095-6.

Grandgenett, N.F. (2008). Perhaps a matter of imagination: Technological pedagogical content knowledge in mathematics education. Chapter 7, Koehler & Mishra, (Eds), *The Handbook of Technological Pedagogical Content Knowledge for Teaching*. New York: Routledge Press.

Heid, M. K. (2005). Technology in mathematics education: Tapping into visions of the future. In W. J. Masalski & P. C. Elliott (Eds.), *Technology-supported mathematics learning environments, the sixty-seventh yearbook of the National Council of Teachers of Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Kaser, J., Bourexis, P., Loucks-Horsley, S., & Raizen, S. (1999) *Enhancing Program Quality in Science and Mathematics*. Thousand Oaks, CA: Corwin Press.

Koehler, M.J. and Mishra, P. (2008). Introducing TPCK. Chapter 1, Koehler & Mishra, (Eds), *The Handbook of Technological Pedagogical Content Knowledge for Teaching*. New York: Routledge Press.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, Virginia: NCTM. Accessible at http://www.nctm.org/.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, Virginia: NCTM. Accessible at http://www.nctm.org/.

National Council of Teachers of Mathematics. (1999). Constructivist views on the teaching and learning of mathematics. In R. Davis, C. Maher, & N. Noddings (Eds.), *Monograph Number 4, Journal for Research in Education.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2003). The use of technology in the learning and teaching of mathematics. *NCTM position statement*, published October, 2003. Accessible at http://standards.nctm.org/.

National Council of Teachers of Mathematics. (2005). Technology-Supported Learning Mathematics Environments. *NCTM's 67th Yearbook*, edited by W.J. Masalski and P.C. Elliott. Reston, Virginia: NCTM.

Priwer, S., Phillips, C. (2003). *The everything Einstein book.* Avon, MA: Adams Media Corporation.

Putnam, A. R. (2002). Problem-based teaching and learning in technology education. Research report submitted to Educational Resources Clearinghouse, ED 465039.

National Science Foundation. (1996). *Shaping the future: New expectations for undergraduate education in science, mathematics, engineering, and technology*, Washington, D.C., 1996.

Peterson, I. (1988). *The mathematical tourist: Snapshots of modern mathematics*. New York, New York: Freeman and Company Publishers.

Reece, G. C., Dick, J., Dildine, J. P., Smith, K., Storaasli, M., Travers, K. J., Wotal, S., & Zygas, D. (2005). Engaging students in authentic mathematics activities through calculators and small robots. In Masalski, W.J. and Elliott, P.C. (Eds.) *Technology-supported mathematics learning environments*, the Sixty-Seventh Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Rosen, K.H. (1999). *Discrete Mathematics and Its Applications*. San Francisco, CA: McGraw Hill.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15 (2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review,* 57 (1). 1-22.

Sinclair, N., Crespo, S. (2006). Learning mathematics in dynamic computer environments. *Teaching Children Mathematics*, 12(9), 437-444.

Steen, L.A. (1988). The science of patterns. *Science*, 240, 29.

United States Department of Education. (2005). *The secretary's fourth annual report on teacher quality, from the office of Postsecondary Education*. Jessup, Maryland: Educational Publications Center, U.S. Department of Education.